The CX-Space: A Unified Paradigm for Grasping Using Multifingered Hands

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***Abstract*— Grasp planning is formulated as a unified problem, fusing the previously decoupled subproblems of locating and matching contacts on the hand and on the payload. To achieve this, the problem is formulated in the CX-space which is denoted as the Cartesian product of the hand's configuration space (C-space), and a space of relative poses between the hand and the payload (X-space). This approach alleviates some difficulties which are unresolved when a search is performed in the hand's C-space alone. A number of kinematic aspects of grasping are addressed by associating them with sets in the CX-space. Solving for a desired grasp then requires identifying a CX-space configuration which lies in the intersection of these sets. Rather than recommending a search technique, this paper presents the framework for this novel formulation. As a demonstration, portions of the CX-space associated with a simulated Utah-MIT hand and a spherical payload are examined.**

I. Introduction

Automated grasping is a central problem in robotic manipulation of objects. In unstructured environments, rather than using task specific end-effectors and tools with prespecified mating features for grasping payloads, system designers may opt to use generic and adaptable interfaces. The human hand has had a track record of millions of years of successfully grasping objects with various shapes and sizes. Therefore, a number of biologically inspired anthropomorphic hands have been developed over the past several decades. These mechanical hands pose a grasping challenge because constraining the 6 degrees of movement of the payload in 3D space requires a careful selection of hand configurations from among the high dimensional space of configurations.

Classical approaches to the problem of grasp automation involve breaking it up into two stages: (1) determining the contact locations and loads on the payload which result in a desired grasp property such as wrench closure, and (2) configuring the robot's hand to make contact with the payload at those locations, and apply the desired loads [1], [4]. In some cases, the standard analyses and the initial solutions they provide do not yield a satisfactory result.

Often, an exhaustive Stage 1 study in which the continuum of solutions for a given number of contacts, as well as the more expansive search of all solutions for all possible numbers of contacts, is quite infeasible. A more palatable approach is to make a selection based on all the available information during Stage 1, which excludes the information about the robot's hand, and then proceed to Stage 2. For a given Stage 1 choice, there may be zero or a multitude of Stage 2 solutions, some of which may suffer from peripheral problems associated with the hand's workspace reach, joint limits, singularities, self-collisions, obstacle avoidance, and so forth. In the event these problems are encountered during Stage 2, the analysis may require to be reverted back for a different Stage 1 choice.

Solving Stage 2 involves an inverse kinematics approach for mapping the hand contacts from (a) their local hand frame poses to (b) their desired global frame poses on the payload, producing (c) a corresponding set of hand joint values. Furthermore, it is necessary for these contacts to be capable of applying the necessary loads. Before applying these inverse kinematics analyses, one challenge is that of how to select targets on the hand to match the targets on the payload. The hand targets may be solely on the fingertips, or they may be distributed throughout the fingers and the palm. There may even be multiple contacts on some fingers, or even some segments of fingers. As already mentioned, the hand's workspace reach, joint limits, singularities, self-collisions, and so forth must be accommodated. Addressing all of these concerns separately in two stages seems nonoptimal, and in practice, heuristic algorithms may be required for each new category of robot hands. It would be difficult to devise these strategies to comprehensively handle all situations for a generic hand with an unknown number and arrangement of fingers and joints.

These issues suggest that there is some fundamental information which is being excluded, and therefore calls for a unified approach which provides a coherent foundation for both Stages 1 and 2, along with how to handle their seam. This paper proposes working in a CX-space which is a hybrid of the hand's configuration space (C-space) and the relative hand-payload's pose space (X-space). The CX-space configurations parameterize *both* the hand and the payload, which were previously addressed in two separate stages. Note that this paper addresses only the kinematic and geometric characteristics of these stages, i.e. the contact positions on both the payload and the hand. The required loads at these contact locations are not addressed in this paper, but are considered to be a postprocessing step.

Already some researchers have begun working with the hand's C-space. Platt, Fagg, and Grupen defined artificial potential fields in the hand's C-space, and used controllers which descend down their steepest descent directions [8]. However, each of their controllers assumed a prespecified arrangement of contacts, and transitions were handled by a higher level mechanism. Yashima recognized the need to incorporate the payload's pose into the configuration, but does not formalize it or rigorously analyze its implications [11].

The contributions of this paper are as follows:

- The concept of the CX-space, a hybrid space of hand joints and payload poses, is formally presented and advocated as a leap above and beyond either the classical 2-stage approach, or formulations in the hand's C-space alone.
- A methodology is described for how this CX-space formulation can accommodate a number of grasp kinematics issues involving the hand's workspace reach, joint limits, singularities, self-collisions, avoidance of other obstacles, and achieving adequate or optimal grasp quality.
- The presented concepts are general enough to apply to point and soft contacts, although for the sake of simplicity, point contacts are used in the simulations.
- Furthermore, one of the strengths of the proposed approach is its seamless and smooth handling of transitions between grasps involving hand-payload contacts of various numbers, combinations, locations, shapes, and sizes.
- This approach may also serve as an extensible foundation for (a) nonclosure grasps such as pushing, for (b) simultaneously grasping multiple payloads such as a screwdriver and a wrench, with a single hand, and for (c) grasp gaiting for manipulating a payload while continually maintaining a desired grasp characteristic such as wrench closure. Although these capabilities are not demonstrated in this work, this paper provides a useful foundation for how they *can conceivably* be implemented.

This paper is organized as follows: Section II presents the structure of the CX-space and discusses some implications. Section III defines various sets of configurations in the CX-space, each associated with an aspect of grasp kinematics. Section IV discusses dimensional decomposition of the various sets which can conceivably be used for improved search performance. Section V presents the results of a simulated grasp of a ball with a Utah-MIT hand, and Section VI closes the paper with concluding remarks.

II. Structure of the CX-Space

The paradigm proposed in this work is the CX-space, which is the product of (1) the hand's configuration space (C-space), and (2) the generalized relative hand-payload pose space (X-space). Note that any number k of payloads being simultaneously grasped can be accommodated by incorporating their pose space in the Cartesian product, as defined in [12].

$$CX = C \; \times \; X_1 \; \times \; \cdots \; \times \; X_k \tag{1}$$

A point in the CX-space contains parameter information from both (1) a hand's joints which can be analyzed in isolation to determine some kinematics aspects such as joint limits and self-collisions, and (2) relative hand-payload pose for all k payloads, which is necessary for determining geometric overlaps, and therefore contacts. Specifying "CX" in "CX-space" helps to distinguish between the two components: The C-space which is completely independent, holonomic, and presumably controllable, and the X-space which is none of these.

In fact, when the hand and a payload are not in contact, the payload is essentially set adrift among the loads applied by the environment. When a payload is grasped, the finger motion affects the relative hand-payload pose via nonholonomic constraints. An arbitrarily small $\delta\theta$ motion along the C-space directions causes a corresponding δX motion along the X-space directions. Therefore, the X-space component of motion is dependent on the C-space component.

This formulation in essence combines both Stages 1 and 2 of the decoupled classical formulations mentioned in Section I. It is interesting to note, that in the decoupled formulation, each subproblem is in the 3D workspace, and involves a variable number of contacts. Furthermore, there are $(n_h!/n_p!)$ permutations of matches between n_h the number of suitable targets on the hand, and n_p those for the payload. So there is much information that is embedded in and must be extracted from the 3D workspace, in which the decoupled approach is formulated.

However with the unified CX-space approach, one is merely concerned with navigating a single point in a generalized $(mk + n)$-dimensional space, constructed from an n-dimensional C-space of the n-DOF hand, and k hand-payload X-spaces, each being m-dimensional. The $(mk + n - m)$ difference in dimensionality between the decoupled and CX-space formulations suggests the enormity of information which is either inadequately or nonoptimally handled in a decoupled approach.

III. Sets of Configurations in the CX-Space

One important advantage of the CX-space formulation is in its simultaneous representation of multiple aspects of grasp kinematics:

1) hand's workspace reach
2) joint limits
3) singularities
4) avoiding self-collisions
5) making hand-payload contact
6) avoiding any other obstacles
7) obtaining adequate or optimal grasp quality

Without loss of generality, the remainder of this paper addresses the nominal situation of manipulating only a single payload. However, by applying the following discussions to the subset $(C \; \times \; X_i), i \in \{1, \cdots, k\}$ of the CX-space, these concepts can be used for $k \geq 1$ payloads. For each of the 7 aspects above, Ψ denotes the set of all configurations in the CX-space that meet a prescribed criteria for a binary inclusion test. These are discussed below:

1) First, Ψ_{reach} is the set of all configurations which are within the workspace reach of the hand, which includes all hand configurations, and hand-payload poses. Thus, this is the entire CX-space, $\Psi_{reach} = \{\forall q\}$. This is mentioned since this issue must be explicitly handled by the decoupled formulation described in Section I.

2) The set $\Psi_{q,\lim}$ of all configurations which are within the hand's joint limits is typically a rectanguloid in the CX-space with walls orthogonal to the principal axes of the C-space.
3) Avoiding singularities is not required for applying the CX-space approach, however it may be useful for circumventing other problems downstream of the grasping solution. In applications, avoiding singularities may be addressed by avoiding configurations whose Jacobian matrices are *too close* to being rank deficient.

   Here, the Jacobian may be defined as the partial derivatives of either the pose of the finger tips, other hand features, or as desired otherwise. A distance to singularity measure d_s along with a minimum threshold value $d_{s,\min}$ would be necessary to define the set $\Psi_{d_s \le d_{s,\min}}$ of all configurations which satisfy $d_s \le d_{s,\min}$, and are therefore too close to singularities. A common distance to singularity measure defined as a product, is $d_s = \prod_i \sigma_i(J)$, where $\sigma_i(J)$ are the singular values of the Jacobian [3]. Since this is independent of the hand-payload pose, $\Psi_{d_s \le d_{s,\min}}$ is invariant in the X-space. The complement to this set, namely $\Psi^{\perp}_{d_s \le d_{s,\min}}$ contains all CX-space configurations which adequately avoid singularities.
4) The hand must never be in self-collision, therefore the set Ψ_{self} of all configurations associated with self-collisions must be avoided. Set Ψ_{self} is invariant in the X-space.

Proposition 1: A surface contact in the workspace between the hand and an object is mapped to the surface of their collision set in the CX-space.

Proof: Suppose q_o is a point in the strict interior of the collision set in the CX-space. Then there exists a ball surrounding q_o defined by $(q_o + \delta q)$ such that $\|\delta q\| \le \epsilon$, which remains in the interior of the collision set. Since the $\|\delta q\| \le \epsilon$ of joint motion keeps the two in contact, q_o must map to a configuration of the hand penetrating the surface of the object in the workspace. A similar argument for q_o being a point in the strict exterior of the collision set in the CX-space, shows q_o being mapped to a configuration of the hand and the object being separated in the workspace. The converse of these arguments can be used to show that the strict interiors map from the workspace to the CX-space, as do the strict exteriors. Therefore, the boundary of the collision set in the CX-space is a bijective map to a nonpenetrative surface contact in the workspace, and vice versa. Q.E.D.

5) A solution grasp necessitates that the hand is in contact (collision) with the payload. Therefore, the solution must be confined to the set Ψ_{pyld} of all CX-space configurations for which the hand and the payload are in collision. Note that Ψ_{pyld} has been the first set so far, which is influenced by the X-space component. As an aside, a CX-space configuration which is not a part of Ψ_{pyld} has the physical meaning that the hand has let go of the payload. Therefore, in the complement set $\Psi^{\perp}_{pyld}$, the payload is set adrift in the environment, and the CX-space configuration may drift along the X-space directions as a function of time.

   When considering ideally hard objects in the workspace, the hand-payload contact is always nonpenetrative. Therefore from Proposition 1, the set of all configurations with nonpenetrative hand-payload contact is on the surface of the hand-payload collision set, i.e. $\partial(\Psi_{pyld})$, where $\partial(\cdot)$ denotes the CX-space surface extraction operator. Soft contacts would involve the inclusion of all configurations which are in Ψ_{pyld} at or beneath the surface with the penetration distance in the workspace being dependent on the applied force, contact area, and contact stiffness. For the rest of this paper, only ideally hard objects are considered.

   In general, Ψ_{pyld} may be disjoint, i.e. $\Psi_{pyld} = \cup\, \Psi_{pyld,j}$, where $\Psi_{pyld,j}$ is a connected subset. Therefore during grasp gaiting operations, in order to avoid releasing the payload, the entire path from start to goal must be completely confined to the boundary of a single connected subset $\partial(\Psi_{pyld,j})$.
6) If there are incidental obstacles to be avoided, other than the payload and the hand, they can be mapped from the workspace to the C-space. Since these obstacles exclude the payload, their CX-space mapping Ψ_{obst}, is invariant in the X-space. Furthermore, if they are moving, then this mapping is time varying.
7) It is useful to have a quantitative measure of grasps, so that for a given application, one can be objectively chosen over another. This measure is denoted as the *grasp quality* Q, where higher values are more desirable. For a given grasping application, it is assumed that a lowest usable grasp quality value $Q_{\min}$ can be selected by the user, defining the set of configurations $\Psi_{Q \ge Q_{\min}}$, as those which satisfy $Q \ge Q_{\min}$. Alternatively, if the *best* grasp is to be chosen, then an optimal grasp exhibits the greatest grasp quality, i.e. $Q = \max(Q)$. This Q may be comprised of measures associated with wrench closure or application specific metrics.

   The former, *wrench closure*, is obtainable by determining the locations and normals of all geometric interferences, or *contacts* between the hand and a payload. These interferences or contacts can be simplified as point contacts, or represented in a more sophisticated way as a soft contact having the shape of the geometric interference. In the presence of friction, a convex hull of the friction cones in the wrench space can be used to quantify the degree of closure in the wrench space [6], [7]. These results are functions of the hand kinematics and payload geometry and relative pose, which are encoded in a CX-space configuration.

This paper does not advocate one definition of the grasp quality Q over another. In fact, it is conceivable that by introducing a measurement bias for the origin of the wrench space, even nonclosure grasps such as those used for pushing, could be accommodated.

Ultimately, any grasp issue can be accommodated using CX-space set inclusions, as long as the issue can be completely characterized by (a) the hand kinematics, (b) payload geometry, and (c) hand-payload relative pose. From the previous discussion, the set Ψ^* of all solution configurations to the grasping problem must satisfy the following relationship:

$$\begin{aligned}\Psi^* \quad = \quad & \Psi_{q,\lim} \bigcap \Psi^{\perp}_{d_s \leq d_{s,\min}} \bigcap \Psi^{\perp}_{self} \bigcap \\ & \partial(\Psi_{pyld}) \bigcap \Psi^{\perp}_{obst} \bigcap \Psi_{Q \geq Q_{\min}} \end{aligned} \quad (2)$$

where $\Psi_{reach} = \{\forall q\}$ was omitted for brevity. If an optimal grasp is sought, then the solution q^* obeys:

$$Q(q^*) = \max_{q \in \Psi^*}(Q(q)) \quad (3)$$

where in this case Ψ^* is defined without the $\Psi_{Q \geq Q_{\min}}$ set. Also note that if the above pertains to a grasp gait, then $\partial(\Psi_{pyld})$ in the above should be replaced by $\partial(\Psi_{pyld,j})$, where $\Psi_{pyld,j}$ is the connected subset of Ψ_{pyld} which contains the start of the gait.

IV. Dimensional Decomposition of the CX-Space

The CX-space structure along with all the sets defined above, address multiple aspects of the grasping problem, simultaneously. Once the final admissible set Ψ^* has been identified, a solution may be selected. Depending on the type of grasping problem, i.e. adequate grasp, optimal grasp, or grasp gait, other calculations may be required for selecting from the configurations in Ψ^*.

A preclusive problem for searching in high dimensional spaces is that of the required computation space (memory) and computation time (number of math operations). The requirement of these resources can sometimes grow exponentially with the search space dimension. Therefore, it would be advantageous to subdivide a search in a high-dimensional space into an equivalent set of searches, each in a lower dimensional space. Similar decomposition strategies have appeared elsewhere in literature [10]. Fortuitously, the structure of several sets discussed in Section III facilitate this divide and conquer strategy. Note that in the following discussion, all hand joints are assumed to belong exclusively to a single finger. However, if there are joints which are common to multiple fingers, i.e. palm joints, then these joints should be included in the set of joints $\{\theta\}_i$ associated with each of the i-th or the parent fingers. The following observations are instrumental in reducing the computational complexity of CX-space formulations:

- Finger Forward Kinematics: The forward kinematics of any point on a given i-th finger is dependent only on the joints of that finger and any other more proximal joints, denoted as $\{\theta\}_i$, and is independent of the joints of all other unrelated fingers $\{\theta\}_i^{\perp}$. Therefore, any calculation which is purely based on hand kinematics can be decoupled and be treated in the subspace of the CX-space associated with $\{\theta\}_i$.
- The set Ψ_{self} of configurations in self-collision is driven purely by the hand's forward kinematics. Clearly, the collision state of a finger with itself, and that of two fingers with each other are independent of unrelated fingers. Therefore,

$$\Psi_{self} = \left(\bigcup_i \Psi_{self,i}\right) \bigcup \left(\bigcup_{i,j,i\neq j} \Psi_{self,ij}\right) \quad (4)$$

  where $\Psi_{self,i}$ is the set of CX-space configurations for which finger i is in self-collision, and $\Psi_{self,ij}$ is the set of configurations for which fingers i and j are in collision. Therefore, $\Psi_{self,i}$ and $\Psi_{self,ij}$ are invariant with the joints of unrelated fingers, and with the X-space. If there are n_f fingers, the number of pair-wise subsets $\Psi_{self,ij}$ to consider is $(n_f!)/(2(n_f-2)!)$.

  As an example, self-collisions for the Utah-MIT hand, which has 4 fingers each with 4 DOF, can be decomposed into the union of finger self-collisions and finger pair collisions. For finger self-collisions, only the 4 DOF of a given finger are relevant, and for pair-wise finger collisions, only the 8 DOF of a finger pair are relevant. In all cases, the poses in the X-space are irrelevant.
- The set Ψ_{pyld} of configurations which produce a hand-payload collision is dependent both on hand joint variables and on the relative hand-payload pose. However, the collisions between the i-th finger and the payload, and hence $\Psi_{pyld,i}$, are independent of all other fingers:

$$\Psi_{pyld} = \bigcup_i \Psi_{pyld,i} \quad (5)$$

  From (2), the boundary $\partial(\Psi_{pyld})$ is of great interest.
- Similar to the previous arguments, the set Ψ_{obst} of configurations which produce collisions between the hand and any existing obstacles can be decomposed into its constituent subsets as

$$\Psi_{obst} = \bigcup_i \Psi_{obst,i} \quad (6)$$

  where i indexes a finger. Subsets $\Psi_{obst,i}$ are invariant with respect to joints in unrelated fingers, and with values in the X-space. However, they may be time varying.

The computational complexity of searching the CX-space for a particular grasp solution can be reduced significantly by performing the searches in fewer dimensional subspaces. To illustrate the difference, the dimensions of the search spaces with and without decomposition are compared. For a single payload task, the dimension of the whole CX-space is $dim(\Psi) = (n+m)$, where n is the DOF of the hand, m is the dimension of the relative hand-payload pose in the workspace. If instead, the search is carried out in the component subspaces as was described above, the search space dimensions become

$$dim(\Psi_{self,ij}) = n_i + n_j \quad (7)$$

$$dim(\Psi_{self,i}) = n_i \quad (8)$$

$$dim(\Psi_{obst,i}) = n_i \quad (9)$$

$$dim(\Psi_{pyld,i}) = n_i + m \quad (10)$$

where n_i is the DOF of finger i. Therefore, the largest dimension of these component searches is

$$\max(dim(\cdots)) = \max(n_i) + \max(\max_{i \neq j}(n_j), m) \qquad (11)$$

As an example, consider the Utah-MIT hand which has 4 fingers, each with 4 DOF, in a 3D workspace (6D X-space). The dimension of the whole CX-space is 22, while the largest dimension of search subspaces is 10. Clearly, there is good savings potential by operating in 10D search spaces instead of 22D spaces.

V. Simulation Results

This paper has proposed a framework for solving grasp kinematics problems. Although a search technique must be used to iteratively arrive at a desired solution, rather than presenting search results, this section presents visualizations of the discussed sets in the CX-space. For brevity, only three of the discussed sets are demonstrated in this section: hand-payload contacts Ψ_{pyld}, self-collisions Ψ_{self}, and adequate grasp quality $\Psi_{Q \geq Q_{\min}}$. Note that $\Psi_{reach} = \{\forall q\}$ is inherently accommodated, and the example uses configurations which adhere to $\Psi_{q,\lim}$. From (2), a solution grasp q^* is one which is simultaneously included on the boundary of the hand-payload contacts $q^* \in \partial(\Psi_{pyld})$, while being *excluded* from self-collisions $q^* \in \Psi_{self}^{\perp}$, and while having adequate grasp quality $q^* \in \Psi_{Q \geq Q_{\min}}$.

These concepts have been implemented in software for a generic multifingered hand and payload, both represented as polytopes. The following is a simulation applied for the Utah-MIT hand and a spherical payload. The Utah-MIT hand has 4 identical fingers, each with 4 joints: a roll joint at the base, along with 3 sequential pitch joints further out. These are denoted as the proximal, medial, and distal pitch joints. The roll joint has a range of motion of $[-60°, 60°]$, while the pitch joints have $[0°, 90°]$.

In order to obtain 2D and 3D visualizations, a simple case with reduced dimensionality must be presented. A spherical payload with a frozen hand-payload pose is used, reducing the dimension of the X-space to 0. From (4)-(5), Ψ_{self} and Ψ_{pyld} are based on their components for the isolated fingers and finger pairs. Therefore, the objective of the following example is to grasp the sphere using a single finger pair, i.e. just the thumb and the index finger, eliminating the 8 DOF associated with the other two fingers. In order to further reduce the problem's dimension, each finger's roll joint at the base, and distal pitch joint, are both held frozen. So this scenario has 4 hand DOF, i.e. the proximal and medial pitch joints of both the thumb and the index fingers. In order to intentionally create the possibility of self-collisions, the thumb's roll joint is frozen at $-13°$. All other hand joints are frozen at $0°$. The spherical payload's position is also frozen at $[5, 0, 5]$ cm relative to the palm's coordinate frame, and since it's a sphere, its orientation is inconsequential.

Figure 1 illustrates the contact between the thumb and the payload. From the caption, the two axes represent the CX-subspace spanned by the two pitch joints of the thumb.

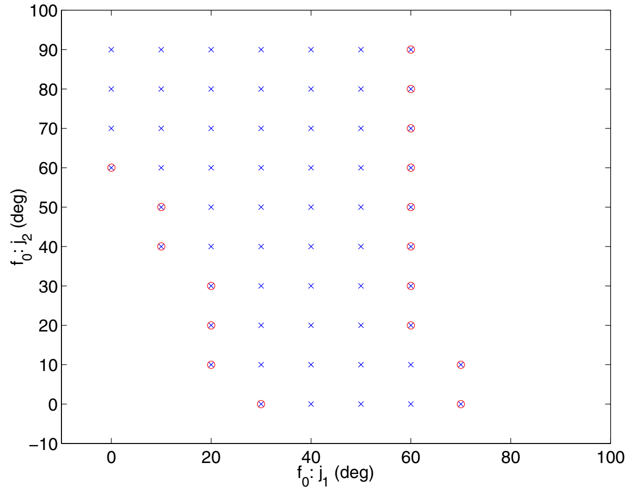

Fig. 1. Collisions states for a simulated Utah-MIT hand's thumb (f_0), with the spherical ball. The two pitch joint variables are shown as $(f_0 : j_1)$ and $(f_0 : j_2)$ for the proximal and medial pitch joints, respectively. The "x"s indicate collisions, and the "o"s indicate the finger-payload boundary contact. The boundary on the left represents contact on the palm side, while the boundary on the right is on the fingernail side.

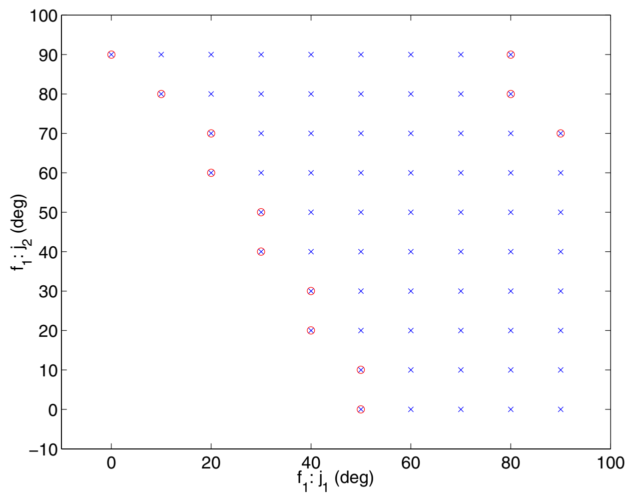

Fig. 2. Collisions states for a simulated Utah-MIT hand's index finger (f_1) with the spherical ball. The symbolism is similar to that in Figure 1.

The set $\Psi_{pyld,0}$ of thumb-payload contacts is represented by the "x"s, and its boundaries by "o"s. The boundary on the $(j_1 = 0°, j_2 = 0°)$ side indicates contact on the palm side of the thumb, while the boundary on the $(j_1 = 90°, j_2 = 90°)$ side indicates contact on the fingernail side of the thumb. Figure 2 illustrates the contact between the index finger and the payload. All symbolism is presented similarly for the set $\Psi_{pyld,1}$ of index finger with payload contacts.

The first step is to identify the configurations which are associated with the boundary of the hand-payload contact. Typically, all boundary conditions should be examined. However for brevity, the cases of single finger to payload contacts are omitted since they obviously will not produce wrench closure grasps. Therefore, only double finger to payload contacts are

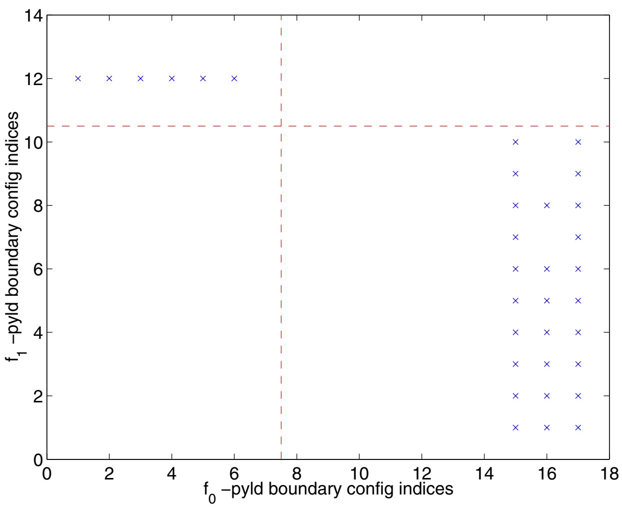

Fig. 3. Illustrated are thumb - index finger self-collision states among all possible hand-payload contact configurations for a simulated Utah-MIT hand. The horizontal axis lists the ordered sequence of finger 0 - payload contacts, and the vertical axis lists those for finger 1 - payload contacts. The ordering is left-to-right and top-to-bottom for the boundary configurations in Figures 1-2. The palm, then fingernail side contacts of the thumb are represented by the areas to the left, then right of the vertical dashed line, respectively. Similarly, the palm, then fingernail side contacts of the index finger are represented by the areas below, then above the horizontal dashed line, respectively.

considered. From the 100 elements in each figure, there are 17 boundary elements from Figure 1 and 13 from Figure 2. So in all there are $(17 \times 13 = 221)$ configurations which have both fingers touching the surface of the payload.

Next the self-collisions are considered. Unfortunately, since this involves 2 DOF from the thumb and 2 from the index finger, their total of 4 DOF are difficult to visualize directly. Therefore, Figure 3 plots the locations of self-collisions between the thumb and the index finger, but only for the relevant configurations, i.e. those with finger-payload boundary contact. The axes represent ordered indices of the configurations which are marked by "o"s in Figures 1-2. From Figure 3's caption, the lower left quadrant contains all the configurations with finger-payload contact on the palm side for both fingers. All of these configurations are seen to be free from self-collisions.

So far, among the 10,000 configurations considered, 221 configurations are members of $\partial(\Psi_{pyld})$ (with both fingers in payload contact), and among them, 188 are *not* in self-collision. The next step is to consider grasp qualities. Note that the CX-space formulation being presented does not impose any restrictions on the grasp quality measurement scheme. The grasp quality in this work is a variant of that presented by Ferrari and Canny [6], with the exception that for the sake of computational speed, only forces are considered, not torques.

Figure 4 plots the sorted grasp qualities for the 188 configurations that have made it this far. The leftmost 31 configurations have a grasp quality of 0, meaning that there is no force closure. Most of these involve configurations where payload contact is made on the fingernail side of both fingers. For this example, without having a prespecified $Q_{\min}$, any

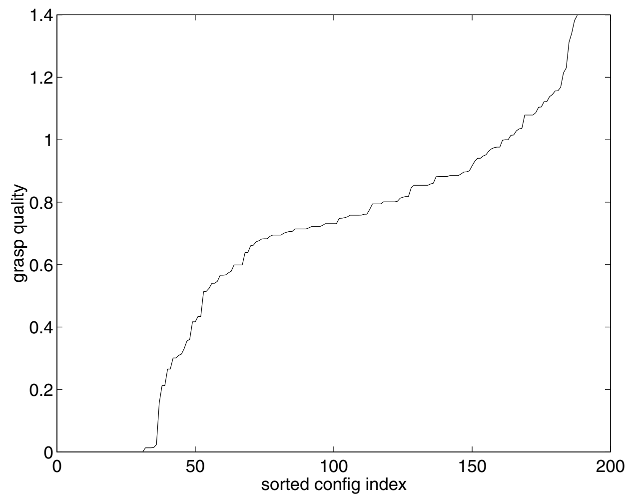

Fig. 4. Shown are the sorted grasp qualities of the 188 configurations which are on the finger-payload boundaries for both fingers, and are free form self-collisions between the two fingers.

configuration with grasp qualities near $\max(Q)$ would be appropriate.

By visual inspection, the configuration with the fourth best grasp quality seems to represent the most robust grasp from among the top 10. Figure 5 illustrates this configuration. It is interesting that although the described selection mechanism numerically leads to a number of solutions in order of optimality, the visually optimal solution is not the number one choice identified numerically. It is hypothesized that this flaw is not a shortcoming of the CX-space framework, and instead is due to the approximations used in the grasp quality evaluation. One limiting factor may be the fact that point contacts are used rather than soft contacts which would accommodate variable shapes and sizes to indicate true geometric overlaps. Another may be that the examined configurations are in a rectangular grid rather than at the hand-payload boundary. Presumably, this causes polytope penetrations that are not physically realizable, and hence corrupts the wrench space convex hull used for the grasp quality measure. In any case, despite the issues with the grasp quality implementation, the CX-space formulation provided a framework for examining 10,000 configurations and selected several good grasp choices from among them. The extra step of applying human judgement to the top few choices led to the good grasp configuration in Figure 5.

Figure 6 shows the numbers of hand-payload contacts for the same sorted sequence of configurations as in Figure 4. This figure shows a very important point: this CX-space formulation seamlessly accommodates variations in the numbers of hand-payload contacts. It is not necessary to group analysis runs based on prespecified numbers of hand-payload contacts, like in many other approaches. Another important point not illustrated here is that this CX-space formulation also seamlessly accommodates variations in other contact parameters, such as contact locations on the hand.

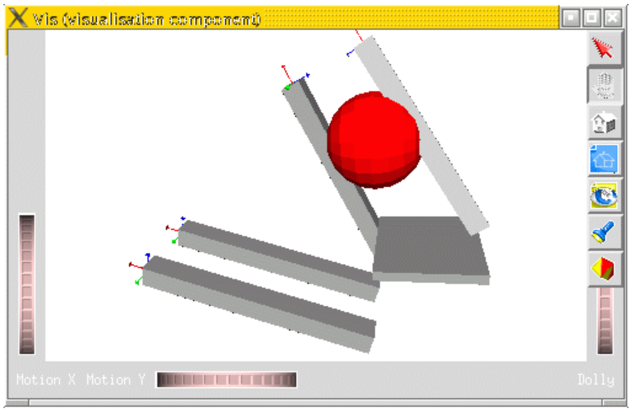

Fig. 5. Shown is the configuration with the fourth highest grasp quality plotted in Figure 4.

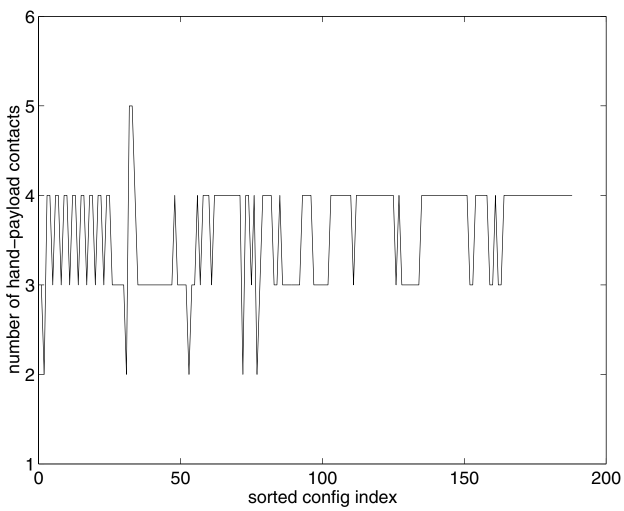

Fig. 6. Shown are the number of hand-payload contacts for the same sorted sequence of 188 configurations in Figure 4.

VI. Concluding Remarks and Future Work

This paper has presented the CX-space paradigm for formulating grasping problems in a generic fashion, that accommodates hands with a variety of numbers and arrangements of fingers. This formulation can also accommodate payloads with a variety of shapes and sizes, and is therefore very general. In this paradigm, several aspects of grasp kinematics can be addressed by identifying sets in the CX-space whose component configurations pass binary inclusion tests. The solution set Ψ^*, whose configurations must pass all tests, is then the intersection of all inclusion sets, as in (2). An adequate grasp is any CX-space configuration from Ψ^*.

The kinematic aspects of grasping that were addressed in this paper, include the hand's workspace reach, joint limits, singularities, self-collisions, avoidance of other incidental obstacles, and achieving adequate or optimal grasp quality. For the sake of efficiency (and in this case visualization), some of these sets can be dimensionally decomposed exposing the minimal set of parameters from among the hand joints and hand-payload relative pose, which influence the kinematic issue being addressed by the set.

Through a simple example of a two fingered grasp by a Utah-MIT hand, this first paper has demonstrated this formulation's accommodation of the hand's workspace reach, joint limits, self-collisions, hand-payload contact, and grasp quality.

Beyond the presented simulation, the proposed CX-space formulation has shed light into a number of grasping applications, which are to be demonstrated in future efforts. The first step is to apply a search technique in this CX-space paradigm for obtaining adequate or optimal grasps. This CX-space formulation can accommodate a variety of grasp quality definitions. Therefore, a possible next step is to define the grasp quality to accommodate soft contacts. The CX-space seamlessly and smoothly handles grasps with various numbers, combinations, locations, shapes, and sizes of hand-payload contacts. The CX-space paradigm has also been formulated to accommodate multiple simultaneous payloads. Therefore, this could be another interesting grasping problem. Finally, the challenging problem of grasp gaiting can conceivably be formulated nicely using the CX-space, by considering paths which remain entirely within a single connected (non-disjoint) subset within Ψ^*, as defined by (2).

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